- **The course** focuses on the descriptive mathematical foundations of Convex Optimization. Convex Optimization is a “solvable case” in Mathematical Programming – under mild computability and boundedness assumptions; one can approximate global solutions to convex optimization problems to whatever high accuracy with reasonable computational effort.

Mathematics of Convex optimization is composed of (a) *descriptive foundations* (existence of optimal solutions, optimality conditions, duality, etc.), (b) modeling (techniques allowing for posing the problem of interest as a convex optimization program), and (c) *operational toolbox* --- algorithms. Traditionally, Mathematical Programming/Conex Optimization courses focus on algorithms and present descriptive foundations of Optimization at a minimal level sufficient to serve the needs of designing and analyzing the algorithms. This course is different: its emphasis is on descriptive foundations and to a lesser extent – on modeling, with the algorithmic component reduced to a brief “executive summary” of Interior Point Methods – the state-of-the-art techniques for processing “well-structured” convex programs. Let me start with a brief historical excurse to motivate the emphasis on descriptive foundations.

Linear Programming (the “starting point” and one of the cornerstones of Convex Optimization) was discovered in the late 1940’s and from the very beginning was equipped with a mighty solution algorithm, the *Dantzig’s Simplex method,* making LP a “working entity,” not a wishful thinking. For over 40 years, the Simplex method was the LP algorithm; those working on other LP computational techniques were considered harmless city lunatics. As a result, the Simplex method was the focus of university courses on LP (some professors were even proud that they extracted the LP theory from this algorithm; in my opinion, this is as meaningful as extracting the first principles of Mechanics from the civil engineering manuals). Well, today, the default setting in all commercial LP solvers is Interior Point methods. The morale: In applied Math, the “algorithmic toolbox” is the most important component, as *far as applications are concerned*; at the same time, it is the “most unstable” component: for LP, it was pivoting algorithms during the first ~50 years, is primarily Interior Point Methods in the last 25-30 years, and nobody knows what will the computational “working horse” of LP in 15 years from now. In contrast, the descriptive component of LP was, for all practical purposes, completed in the early 1950s and has remained intact since then.

I believe that as far as university education is concerned, it is important, along with “ready to use” knowledge that will serve the student well immediately upon graduation, to provide the students with “timeless” knowledge of foundations that will serve them well on the span of their entire professional life. The Pythagoras’ Theorem, known as a “practical rule” for at least 3500 years and as a theorem – for about 2300 years, still is as instructive and useful as 2300 years ago; in contrast, how Babylonians, ancient Greeks, and Romans did their Arithmetic, is of absolutely no importance today and is interesting only to those studying the history of civilization.

- **The contents,** aside from the "executive summary" on Interior Point methods, represent what is called Convex Analysis (this is the technical name of the descriptive foundations of Mathematical Programming/Convex Optimization), including

  - Basic results on the geometry of convex functions and sets (definitions, elementary properties, Caratheodory's, Helly's, Krein-Milman,... theorems, subdifferentials, Legendre transform,...)
  - Calculus of convexity
  - Convex optimization problems in Mathematical Programming, cone-constrained, and conic forms (definitions, basic properties, duality, optimality conditions,...)
  - Saddle points and Sion-Kakutani Theorem

To get a complete impression of the descriptive component of the contents, see the course textbook at

https://www.isye.gatech.edu/~nemirovs/KKN_EssMath.pdf

- **Prerequisites.** Formal prerequisites are the most basic Linear Algebra, Calculus, and Real Analysis. The informal (and crucial!) prerequisite is the basic mathematical culture – the ability to comprehend, create, and enjoy rigorous mathematical reasoning. To give an example, the claim "2 x 2 = 5" does not witness the lack of mathematical culture; this is just a miscalculation. In contrast, the claim "2 x 2 = triangle" (believe me, from time to time, I hear in class something like this) does witness the luck of mathematical culture: one should know that under any circumstances, rain or snow, the product of two reals is a real, and not a triangle or a violin.

- **Grading policy**: no obligatory homework, no Mid-term; the grade will be based on the take-home Final Exam.

To complete my informal Syllabus, let me present here my favorite Mathematical joke:

A team flying on a balloon lost their orientation. Suddenly a gust of wind brought them closer to the ground, and they cried to a man they saw below: "Hey, where are we?" After a short pause, they got the answer:" You are on a balloon." The next gust of wind lifted the balloon, and as their journey continued, the leader of the team said: "This was a mathematician. First, he thought before answering. Second, his answer was absolutely correct. Third, it was absolutely useless."

If you fully agree with the leader, then perhaps this course is not your best choice. As for me, I would say that

- To get a helpful answer, ask a proper question: do not ask, “Where are we?”; ask, “What is our location?”
- A useless answer, by definition, cannot be used and is therefore harmless. In contrast, acting based on a seemingly useful incorrect answer, you can get into trouble…

Arkadi Nemirovski