ISYE 8813 Special Topics in OR

Statistical Inference via Convex Optimization

Contents: When speaking about links between Statistics and Optimization, what comes to mind first is the indispensable role played by optimization algorithms in the ``computational toolbox'' of Statistics (think about the numerical implementation of the fundamental Maximum Likelihood method). However, on a second thought, we should conclude that no matter how significant this role could be, the fact that it comes to our mind first primarily reflects the weaknesses of Optimization rather than its strengths; were optimization algorithms which are used in Statistics as efficient and as reliable as, say, Linear Algebra techniques, nobody would think about special links between Statistics and Optimization, just as nobody usually thinks about special links between Statistics and Linear Algebra. When computational, rather than methodological, issues are concerned, we start to think about links of Statistics with Optimization, Linear Algebra, Numerical Analysis, etc. only when computational tools offered to us by these disciplines do not work well and need the attention of experts in these disciplines.

The goal of this course is to present other types of links between Optimization and Statistics, those which have little in common with algorithms and number-crunching. What we are speaking about, are the situations where Optimization theory (theory, not algorithms!) seems to be of methodological value in Statistics, acting as the source of statistical inferences with provably optimal, or nearly so, performance. In this context, we focus on utilizing Convex Programming theory, mainly due to its power, but also due to the desire to end up with inference routines reducing to solving convex optimization problems and thus implementable in a computationally efficient fashion. Therefore, while we do not mention computational issues explicitly, we do remember that at the end of the day we need a number, and in this respect, intrinsically computationally friendly convex optimization models are the first choice.

The three topics we intend to consider are:

A. Sparsity-oriented Compressive Sensing. Here the role of Convex Optimization theory as a creative tool motivating the construction of inference procedures is relatively less important than in the two other topics. This being said, its role is by far non-negligible in the analysis of Compressive Sensing routines (it allows, e.g., to derive from ``first principles'' the necessary and sufficient conditions for the validity of L_1 recovery).

B. Pairwise and Multiple Hypothesis Testing, including sequential tests, estimation of

linear functionals, and some rudimentary design of experiments.

C. Recovery of signals from noisy observations of their linear images.

B and **C** are the topics where, as of now, the approaches we present in this course appear to be the most successful.

The exposition does *not* require prior knowledge of Statistics and Optimization; as far as these disciplines are concerned, all necessary facts and concepts are incorporated into the course. The actual prerequisites are basic Calculus, Probability, and Linear Algebra.

Grading policy: Final take home exam 100%

Links:

Lecture Notes https://www2.isye.gatech.edu/~nemirovs/StatOptLNFall2023NoSol.pdf

Transparencies https://www2.isye.gatech.edu/~nemirovs/StatOptTRFall2023.pdf